THE NIKODYM PROPERTY OF BOOLEAN ALGEBRAS AND CARDINAL INVARIANTS OF THE CONTINUUM

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A common issue in measure theory is to decide whether a sequence of Radon measures on a compact space convergent with respect to some topology is convergent with respect to a finer one. E.g. Nikodym proved that if K is the Stone space of a σ-complete Boolean algebra $\mathcal{A}$ (i.e. K is extremely disconnected), then for any sequence (μ_n) of Radon measures on K the following holds:

if $\mu_n(A) \to 0$ for every clopen $A \subseteq K$ (*pointwise convergence on* $\mathcal{A}$), then
$\int_K f d\mu_n \to 0$ for every $f \in C(K)$ (*weak* convergence in* $C(K)^*$);

On the other hand, if K is an infinite compact space containing a non-trivial convergent sequence (e.g. K is second-countable), then Nikodym's theorem does not hold.

Since every infinite σ-complete Boolean algebra has cardinality at least 2^ω and every infinite second-countable totally disconnected compact space contains a convergent sequence, it follows that the minimal cardinality $\mathfrak{n}$ of a Boolean algebra with the Nikodym property (i.e. such for which Nikodym's theorem holds) is a cardinal characteristics of the continuum. During the first part of my talk I will present the lower bounds for $\mathfrak{n}$ in terms of some classical cardinal characteristics (like $\mathfrak{b}$...). The second (main) part will be devoted to the construction of an algebra with the Nikodym property and small cardinality (consistently less than 2^ω), i.e. to limiting $\mathfrak{n}$ from above. Analysing steps of the construction in terms of classical set-theoretical objects (such as dominating families, selective ultrafilters or non-meager subsets of $\mathbb{R}$) we will show how the cardinality of the obtained algebra is dependent on infinitary combinatorics.

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